

# Comment on “Interface tension of Bose-Einstein condensates” by Bert Van Schaeybroeck, Phys. Rev. A **78**, 023624-9 (2008)

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The purpose of the comment is to point out that the leading term of the Ginzburg-Landau nonanalytical correction to the interface tension of Bose-Einstein condensates with strong segregation and the surface tension of extreme type-I superconductors are described by a common coefficient derived from the universal equation for the phase boundary. The agreement between the numerical value of the coefficients gives a hint that this can be an exact result which deserves to be checked. The outcome will be of interest for physicists working in both fields.

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Recently, the interface tension of the Bose-Einstein condensates attracted significant attention and the commented article<sup>1</sup> is one of the theoretical studies of the problem. The author states that the main result of his paper<sup>1</sup> is the formula for the interface tension Eq. (3), which in the used notations can be rewritten as

$$\gamma_{12} = A^*(\xi_1 + \xi_2)P - B^*P\zeta_0 - \left(\frac{\xi_1}{\xi_2} + \frac{\xi_2}{\xi_1}\right) \left[ \frac{C^*}{(\sqrt{K})^1} + \frac{D^*}{(\sqrt{K})^2} + \dots \right] P\zeta_0. \quad (1)$$

The first coefficient  $A^* = 4\sqrt{2}/3$  coincides with the classical result of Ginzburg-Landau<sup>2</sup> (GL), The second coefficient  $B^* = 4 \times 0.514 \approx 2.06$  also agrees within the accuracy of the numerical calculations with the analytical result<sup>3</sup>

$$d_\tau^2 X(\tau) = X^2(-\tau)X(\tau), \quad X(-\infty) = 0, \quad d_\tau X|_{\tau=+\infty} = 1, \\ B^* \equiv 2^{9/4} \int_{-\infty}^{\infty} (1 - d_\tau X) d_\tau X d\tau, \quad \tau \equiv \hat{z}/2^{1/4}. \quad (2)$$

for the superconductor surface tension<sup>3</sup>

$$\gamma = A^*\xi P - [B^* + \tilde{C}^*\kappa + \tilde{D}^*\kappa^2 + \dots] P\sqrt{\xi\lambda}, \quad \kappa = \frac{\lambda}{\xi}. \quad (3)$$

This numerical coincidence reveals an important new relation between the surface tension of the extreme type-I superconductors and Bose gases with strong segregation which deserves further analysis.

For superconductors,  $\xi$  is the temperature dependent coherence length,  $\lambda$  is the temperature dependent penetration depth,  $P = B_c^2/2\mu_0$ , is the magnetic pressure,  $B_c$  is the temperature dependent critical magnetic field,  $\mu_0$  is the permeability of the vacuum,  $\zeta_0 = \sqrt{\xi\lambda}$ , and  $\kappa$  is the GL parameter. We consider that the coefficient  $B^*$  is common for both superfluids (the adjacent Bose gases and the type-I superconductors) and it is actually the action of an instanton solution of the universal GL equations<sup>2</sup>

$$d_\tau^2 X = Y^2 X, \quad d_\tau^2 Y = X^2 Y, \quad (4)$$

$$X(-\infty) = Y(+\infty) = 0, \quad d_\tau X|_{\tau=\infty} = 1 = -d_\tau Y|_{\tau=-\infty}.$$

We have to consider  $\tau$  as the time for two dimensional motion of a fictitious particle in the potential  $-\frac{1}{2}X^2Y^2$ . The corresponding mechanical problem is depicted in Fig. 5. of Ref. [3]. The further coefficients  $C^*$ ,  $D^*$ ,  $\tilde{C}^*$ , and  $\tilde{D}^*$  of the G-L analytical expansions in power of  $K^{-1/2}$  or  $\kappa$  can be calculated as a perturbation on the instanton action.

For the flat geometry of the domain wall the phase invariance of the superfluid order parameter is irrelevant. The universal constant  $B^* = 2.056347\dots$ , describes the energy of the domain wall for strongly repulsing order parameters in framework of the original Landau theory<sup>4</sup> for the second order phase transitions. For applicability of this result is necessary energy density to have power expansion with respect of order parameters and their first derivatives and the influence of the fluctuations to be small. In the spirit of the Landau theory one and the same mathematical problem can be applied for different physical systems.

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<sup>1</sup> Bert Van Schaeybroeck, “Interfacial tension of Bose-Einstein condensates”, Phys. Rev. A **78**, 023624-9 (2008); addendum **80**, 065601 (2009).

<sup>2</sup> V. L. Ginzburg and L. D. Landau, “On the theory of superconductivity”, Zh. Eksp. Teor. Fys. **20**, 1064 (1950); English translation in: L. D. Landau, Collected papers (Oxford: Pergamon Press, 1965) p. 546.

<sup>3</sup> T. M. Mishonov, “On the theory of twinning plane superconductivity”, Bulg. J. Phys. **15**, 352-367 (1988); “On the theory of type-I superconductor surface tension and twinning-plane-superconductivity”, J. Phys. France **51**, 447-457 (1990).

<sup>4</sup> L. D. Landau, “Theory of phase transformations. I” Zh. Eksp. Teor. Fys. **7**, 19-32 (1937); Phys. Z. Sowjetunion **11**, 26 (1937).